HYDROGASDYNAMICS IN TECHNOLOGICAL PROCESSES

APPLICATION OF THE PARAMETER-EXPANSION METHOD TO CALCULATION OF TWO-PHASE FLOWS IN CHANNELS WITH INJECTION

K. N. Volkov   UDC 532.529:536.24

Velocity and concentration distributions of the condensed phase have been obtained based on the parameter-expansion method. The characteristics of two-phase flows in channels with strong and weak injections have been investigated. The factors exerting an influence on the velocity of nonequilibrium phase motion have been determined; the range of applicability of the solution obtained has been established and its qualitative behavior has been elucidated; the possibility of applying such a solution to calculation of the concentration of the condensed phase has been shown.

Introduction. Metal additions in the form of a highly dispersed powder (mainly aluminum) that enter into the composition of numerous types of modern mixture solid propellants are intended to ensure the required level of energy characteristics and the damping of uncontrolled acoustic oscillations of the working-medium parameters in combustion chambers.

The model of flow of products of solid-propellant disintegration is flow in a channel with a distributed injection, which reflects the most substantial aspect of the process, i.e., the supply of mass from the burning-charge surface. The injection models the burning of the interior channel surface (strong injection) or its thermal destruction (weak injection). The processes associated with the warmup of the propellant and with the disintegration of its components and their chemical reaction occur in a thin surface layer and are disregarded in this model.

In the literature [1–5], much attention is given to the modeling of flows in channels with permeable walls, including the cases where turbulence and the reverse influence of the condensed phase are allowed for. Both the interpenetrating-continuum model [4] and the discrete-trajectory method of test particles [5] are used for description of impurity motion. Many works seek to investigate the stability of flows in channels with permeable walls [6–9].

Modeling of flows in channels with injection calls for modern computing aids in most cases. However, under certain constraints imposed on the particle size and the geometry of the computational domain, one is able to construct exact or approximate solutions convenient for performing evaluations and checking numerical calculations. Evaluation of the range of applicability of such solutions calls for additional investigations in each specific case.

In this work, we consider problems associated with the modeling of two-phase flows in channels with a distributed injection based on the parameter-expansion method. The approach proposed enables us to obtain the distribution of the condensed-phase parameters accurate to the terms linear with respect to the Stokes number. We single out the factors exerting an influence on the velocity of nonequilibrium phase motion and investigate the range of applicability of the solution obtained. The velocity and concentration distributions of the condensed phase in channels with a strong (nonviscous approximation) and weak (creeping flow) injection are given in finite form. We show the possibility of applying such a solution to calculation of the concentration of the condensed phase.

Basic Equations. Let us consider quasideveloped flow of a viscous incompressible fluid in an infinite plane slot of half-width h from both walls of which the fluid is injected at a rate v_w (Fig. 1). We bring the x axis of the

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$$u=0, \quad v=-v_{\mathrm{w}}$$
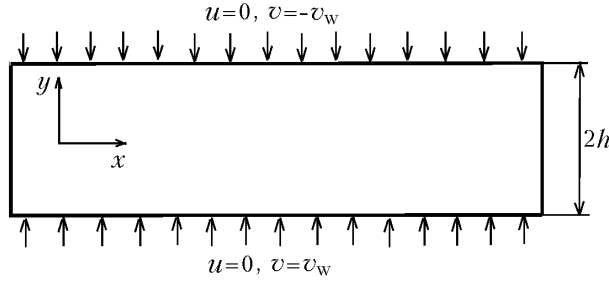
$$u=0, \quad v=v_{\mathrm{w}}$$

Fig. 1. Diagram of flow in a channel with injection.

Cartesian coordinate system into coincidence with the plane of symmetry of the channel. We take the spreading of the fluid to be symmetric about the plane $x = 0$.

The condition of quasidevelopment of flow means that the flow characteristics referred to the maximum velocity in the cross section change with channel length only slightly [2]:

$$\frac{h}{u_{\mathrm{m}}} \left| \frac{du_{\mathrm{m}}}{dx} \right| \ll 1 \ .$$

Such a flow is always established in fairly long channels behind the region of the inlet portion in injection.

Particles represent undeformable spheres of the same diameter; their collisions and reverse influence on the gas are disregarded. Just the hydrodynamic-resistance force is allowed for in the model of interaction of a particle with the carrier flow. The coefficient of resistance of the particle is found from the Stokes law.

We select the channel half-width h as the characteristic scale for variables with the dimensions of length and the injection rate v_{w} for variables with the dimensions of velocity. The continuity and momentum equations for the gas and dispersed phases have the following form:

$$\frac{\partial v_{gk}}{\partial x_k} = 0 \ , \tag{1}$$

$$\rho_g v_{gk} \frac{\partial v_{gi}}{\partial x_k} = -\frac{\partial p}{\partial x_i} + \mu \frac{\partial^2 v_{gi}}{\partial x_k x_k} \ , \tag{2}$$

$$\frac{\partial \rho_p v_{pk}}{\partial x_k} = 0 \ , \tag{3}$$

$$\rho_p v_{pk} \frac{\partial v_{pi}}{\partial x_k} = \frac{1}{\mathrm{Stk}} (v_{gi} - v_{pi}) \ . \tag{4}$$

Summation is made with respect to the double subscripts. The characteristic parameters of the problem are the Reynolds and Stokes numbers:

$$\mathrm{Re} = \frac{\rho_g^{\mathrm{o}} v_{\mathrm{w}} h}{\mu} \ , \quad \mathrm{Stk} = \frac{2}{9} \left(\frac{r_{\mathrm{p}}}{h} \right)^2 \frac{\rho_{\mathrm{p}}^{\mathrm{o}}}{\rho_g^{\mathrm{o}}} \mathrm{Re} \ .$$

First we solve Eqs. (1) and (2) describing the distributions of the parameters of the carrier flow; thereafter we find the concentration and velocity distributions of the dispersed phase by integration of Eqs. (3) and (4).

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Transformation of the Equations. Let us assume that the longitudinal component of the velocity of the gas phase changes linearly along the coordinate x and the transverse component is dependent just on the coordinate y:

$$u_g = xf(y), \quad v_g = g(y). \tag{5}$$

Using (5) and eliminating pressure from the momentum equations in the projections onto the x and y axes with the operation of cross differentiation, we obtain the equation

$$g^{(4)} - \mathrm{Re}\,(gg''' - g'g'') = 0. \tag{6}$$

Boundary conditions for Eq. (6) are set on the plane of symmetry and the channel wall: $g(0) = 0$, $g'(0) = 0$, $g(1) = -1$, and $g'(1) = 0$. The functions f and g are related by the continuity equation ($f = -g'$). When $\mathrm{Re} \to 0$ (weak injection) and $\mathrm{Re} \to \infty$ (strong injection), Eq. (6) has the exact solutions [1, 2]:

$$\mathrm{Re} \to 0 \quad f = \frac{3}{2}\,x\,(1 - y^2), \quad g = -\frac{1}{2}\,y\,(3 - y^2); \tag{7}$$

$$\mathrm{Re} \to \infty \quad f = \frac{\pi}{2}\cos\left(\frac{\pi}{2}\,y\right), \quad g = -\frac{1}{y}\sin\left(\frac{\pi}{2}\,y\right). \tag{8}$$

The solution (8) fairly well describes the velocity distribution for $R > 80$, including the turbulent regime [1–3]. For the dispersed phase, we assume, analogously to (5), that

$$u_p = x\varphi(y), \quad v_p = \psi(y). \tag{9}$$

The concentration of the condensed phase is either calculated based on the assumption of the similarity of the distribution along the longitudinal coordinate

$$\rho_p = \chi(y), \tag{10}$$

or is found in complete form:

$$\rho_p = \rho_p(x, y). \tag{11}$$

With account for (9), the momentum equations for the condensed phase will take the form

$$\psi\varphi' + \varphi^2 - \frac{1}{\mathrm{Stk}}\,(f - \varphi) = 0, \tag{12}$$

$$\psi\psi' - \frac{1}{\mathrm{Stk}}\,(g - \psi) = 0. \tag{13}$$

Boundary conditions are set on the channel wall: $\varphi(1) = 0$ and $\psi(1) = -\omega$.

Qualitative Behavior of the Solution. The velocity of the dispersed phase on the plane of symmetry of the channel is equal to zero. Otherwise there would be a point $y_1 \neq 0$ at which we would have $\psi(y_1) = 0$. However, at the point $y = y_1$, as is easily shown, the right-hand side of Eq. (13) does not satisfy the Lipschitz condition.

Restricting ourselves to the expansion terms of first order of smallness, in the vicinity of the channel's plane of symmetry we may write

$$g(y) = g'(0)\,y + o(y), \quad \psi(y) = \psi_1 y + o(y);$$

after the substitution into (13) we obtain the quadratic equation

$$\text{Stk } \psi_1^2 + \psi_1 + g'(0) = 0 .$$

When 4Stk $g'(0) \leq 1$ the above equation has two real roots:

$$\psi_1 = \frac{-1 \pm \sqrt{1 - 4\text{Stk } g'(0)}}{2\text{Stk}} ,$$

here $\psi_1 = -g'(0)$ for Stk $\to 0$. When 4Stk $g'(0) > 1$ the quadratic equation has two complex conjugate roots, and the solution in self-similar form does not exist.

The time a particle escaping from the lateral channel surface takes to reach the plane of symmetry is evaluated from the relation

$$\int_0^t dt = \int_0^1 \frac{dy}{\psi(y)} .$$

Since $1/\psi$ acts as $1/y$ near the plane of symmetry, the particle does not reach the line $y = 0$ over a finite period.

From Eq. (12) it follows that the longitudinal component of the condensed-phase velocity has its maximum for $y = 0$:

$$\varphi_0 = \frac{-1 + \sqrt{1 + 4\text{Stk } g'(0)}}{2\text{Stk}} .$$

Since we have $\varphi_0 \leq g'(0)$, the maximum velocity of the condensed phase is lower than the corresponding value for the gas.

Parameter-Expansion Method. The motion of a Stokes particle of variable size with allowance for the action of mass forces is described by the equation

$$\frac{d\mathbf{v}_p}{dt} = B (\mathbf{v}_g - \mathbf{v}_p) + \text{Fr} , \tag{14}$$

where $B = 1/(\text{Stk } \delta^2)$ and $\delta = r_p/r_{p0}$. The general solution of (14) is sought in the form of the sum of the general solution of the corresponding homogeneous equation $\hat{\mathbf{v}}_p = \mathbf{C}_1 \exp(-F)$ and the particular solution of the nonhomogeneous equation:

$$\mathbf{v}_p = \hat{\mathbf{v}}_p + \tilde{\mathbf{v}}_p .$$

To find the latter we use the parameter–expansion method [10]:

$$\tilde{\mathbf{v}}_p = \mathbf{v}_g + \sum_{k=1}^{\infty} (-1)^k \text{Stk}^k \frac{d^k \mathbf{v}_g}{dt^k} .$$

Restricting ourselves to the terms of first order of smallness in the expansion, we write the solution of (14) in the form

$$\mathbf{v}_p = \mathbf{C}_1 \exp(-F) + \mathbf{v}_g + \text{Stk} \left(\text{Fr} - \frac{d\mathbf{v}_g}{dt} \right) \delta^2 . \tag{15}$$

Here we have

$$F = \frac{1}{\text{Stk}} \int_0^t \frac{d\tau}{\delta^2(\tau)} ; \quad \mathbf{C}_1 = \mathbf{v}_{p0} - \mathbf{v}_g - \text{Stk} \left[\text{Fr} - \left(\frac{d\mathbf{v}_g}{dt} \right)_0 \right] .$$

Relation (15) shows that the two-phase flow tends to an equilibrium one $(\mathbf{v}_p - \mathbf{v}_g)$ for $\mathrm{Stk} \to 0$. When $\mathrm{Stk} \neq 0$ and $r_p \to 0$ we also obtain that $\mathbf{v}_p - \mathbf{v}_g$. In the final step of evolution, particles come into dynamic equilibrium with the gas phase.

The coordinates of a particle are found by solving the kinematic relation

$$\frac{d\mathbf{r}_p}{dt} = \mathbf{v}_p . \tag{16}$$

Integrating (16) from 0 to t, for $\delta = 1$ we obtain

$$\mathbf{r}_p = \mathbf{C}_2 - \mathbf{C}_1 \, \mathrm{Stk} \, \exp(-t/\mathrm{Stk}) + \mathbf{v}_g t + \mathrm{Stk} \, (\mathrm{Fr} \, t - \mathbf{v}_g) . \tag{17}$$

Since the influence of the exponent decreases with Stk in the second term of the right-hand side of (17), we use the time of equilibrium phase motion as t. The constants of integration $\mathbf{C}_1$ and $\mathbf{C}_2$ are determined from the initial conditions. At the instant of time $t = 0$, the particle is on the channel wall; therefore, we obtain

$$x_p(0) = x_{p0} , \quad y_p(0) = 1 , \quad u_p(0) = 0 , \quad v_p(0) = -\omega .$$

The resulting solution is linear with respect to the Stokes number and satisfactorily describes the trajectories of the dispersed component in channels of small length. When $\mathrm{Stk} \to \infty$ it qualitatively incorrectly reflects the behavior of the particle velocity.

The pattern of motion of a particle of variable size is determined by the parameter $H = t_p/t_b$, equal to the ratio of the time of residence of the particle in the channel t_p to the time of burning of the particle t_b. Conditions for burning of the particle are improved with increase in H. When $H = \infty$ the particle burns out on the channel wall, and it does not burn when $H = 0$. Let us assume that the variation in the particle size is described by the equation [3]

$$r_p = r_{p0} \, (1 - t/t_b)^{1/q} , \quad t \leq t_b .$$

Then for $q = 2$ we find the particle velocity from the relation

$$\mathbf{v}_p = \mathbf{C}_1 \, (1 - Ft)^{1/(H\mathrm{Stk})} + \mathbf{v}_g + (1 - Ft) \left(\mathrm{Fr} - \frac{d\mathbf{v}_g}{dt} \right) \mathrm{Stk} .$$

Velocity of Nonequilibrium Phase Motion. We transform the solution (15) to the form

$$|\mathbf{v}_g - \mathbf{v}_p| = \mathbf{C}_1 \exp(-F) + \mathrm{Stk} \left(\mathrm{Fr} - \frac{d\mathbf{v}_g}{dt} \right) \delta^2 . \tag{18}$$

The first term on the right-hand side of (18) determines the influence of the initial conditions, the second term determines the influence of mass forces, and the third term determines the influence of the gradient nature of flow. The criterion of allowability of the parameter expansion of the particle velocity is the condition of smallness of the degree of velocity nonequilibrium of phases $|\mathbf{v}_g - \mathbf{v}_p| \ll \mathbf{v}_g$.

Equilibrium conditions correspond to the case $\mathbf{C}_1 = 0$. The initial conditions do not influence further particle motion, and the equilibrium initial velocity is found from the relation

$$\mathbf{v}_{p0} = \mathbf{v}_g + \left[\mathrm{Fr} - \left(\frac{d\mathbf{v}_g}{dt} \right)_0 \right] \mathrm{Stk} .$$

The intensity of attenuation of the influence of the initial particle state is determined by the Stokes number, and in the case of motion of a particle of variable mass it is dependent on the law of variation in its size. The influence of mass forces on the velocity of nonequilibrium phase motion (in terms of the Froude number) increases with Stokes number, decreases with δ, and is the most pronounced in low-velocity flow regions. In a gradient-free flow, we have $d\mathbf{v}_g/dt = 0$ and the influence of the gasdynamic factor is absent; for $d\mathbf{v}_g/dt \neq 0$ it grows with Stokes number and decreases with
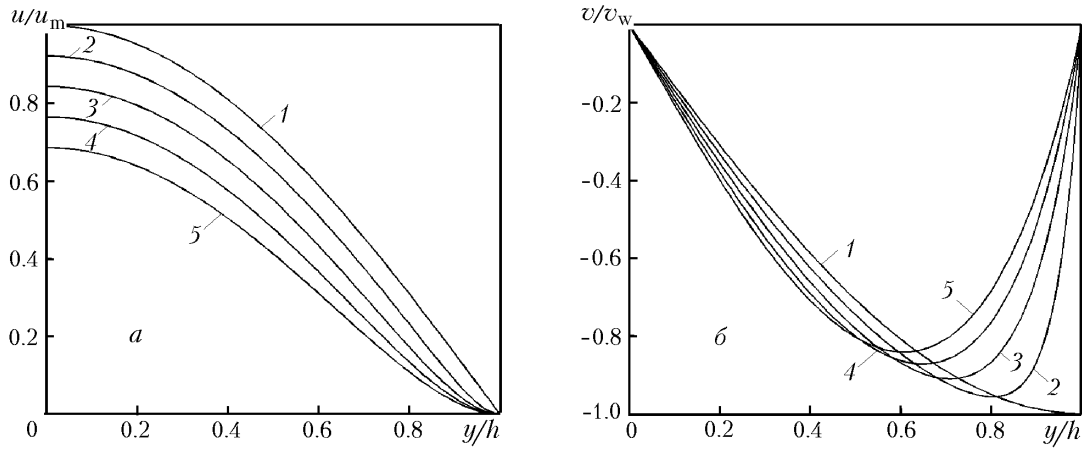
Fig. 2. Distributions of the longitudinal (a) and transverse (b) components of the condensed-phase velocity for Stk = 0.2 (2), 0.4 (3), 0.6 (4), and 0.8 (5). Curve 1 corresponds to the carrier flow (Re $\to \infty$).

δ. The derivative $d\mathbf{v}_g/dt$ is dependent on the geometric shape of the channel and attains the highest value in narrow channels and at the sites of turn of the flow.

Solution in Finite Form. In channels with strong and weak injections, relations (15) and (17) may be represented in finite form.

Solution for the Case of Strong Injection. The coordinates of a fluid particle are found from the relations

$$x = x_0 \cosh^2(t/2), \quad y = \frac{4}{\pi} \arctan\left[\exp\left(-\frac{\pi}{2}t\right)\right].$$

We find the time in equilibrium phase motion:

$$t = -\frac{2}{\pi} \ln\left|\tan\left(\frac{\pi}{4}y\right)\right| = -\frac{1}{\pi} \ln\left|\frac{1 - \cos(\pi y/2)}{1 + \cos(\pi y/2)}\right|.$$

Determining the constants of integration from the initial conditions, we obtain, accurate to the $O(\text{Stk}^2)$ terms, the following relations, yielding the values of the velocity components and the coordinates of the particle at an arbitrary instant of time:

$$u_p = u_g + \frac{\pi^2}{4} \text{Stk}\, x_{p0} \exp(-t/\text{Stk}) - \frac{\pi^2}{4} \text{Stk}\, x_p,$$

$$v_p = v_g + (1 - \omega) \exp(-t/\text{Stk}) - \text{Stk}\left[\frac{\pi}{4} \sin(\pi y_p)\right],$$

$$x_p = x - \frac{\pi}{2} \text{Stk}\, x \cos\left(\frac{\pi}{2}y\right),$$

$$y_p = y - \omega \text{Stk} - (1 - \omega) \text{Stk} \exp(-t/\text{Stk}) + \text{Stk} \sin\left(\frac{\pi}{2}y_p\right).$$

The distributions of the components of the condensed-phase velocity in a channel with a strong injection are given in Fig. 2 for $\omega = 0$. The influence of the initial nonequilibrium of the flow brings about the deformation of the profile of the transverse component of the dispersed-phase velocity near the mass-supply surface. However, by and large the distributions of the transverse component of the condensed-phase velocity differ in a wide range of parameters comparatively slightly.

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Solution for the Case of Weak Injection. The coordinates of a fluid phase here are found from the relations

$$\ln (x/x_0) = -3t + \frac{3}{2} \ln \left[\exp (3t) + \frac{1}{2} \right], \quad y^2 = 3/[2 \exp (3t) + 1] ;$$

the time in equilibrium phase motion is

$$t = \frac{1}{3} \left[\ln \left(|3 - y^2| / y^2 \right) - \ln 2 \right].$$

Determining the constants of integration from the initial conditions, we obtain, accurate to the $O(\mathrm{Stk}^2)$ terms, the following relations, yielding the values of the velocity components and the coordinates of the particle at an arbitrary instant of time:

$$u_{\mathrm{p}} = u_{\mathrm{g}} + 3\mathrm{Stk}\, x_{\mathrm{p0}} \exp (-t/\mathrm{Stk}) - \mathrm{Stk} \left[\frac{9}{4} (1 - y_{\mathrm{p}}^2)^2 + \frac{3}{2} x_{\mathrm{p}} y_{\mathrm{p}}^2 (3 - y_{\mathrm{p}}^2) \right],$$

$$v_{\mathrm{p}} = v_{\mathrm{g}} + (1 - \omega) \exp (-t/\mathrm{Stk}) - \frac{1}{4} \mathrm{Stk}\, y_{\mathrm{p}} (9 - 12 y_{\mathrm{p}}^2 + 3 y_{\mathrm{p}}^4),$$

$$x_{\mathrm{p}} = x - \frac{3}{2} \mathrm{Stk}\, x (1 - y^2),$$

$$y_{\mathrm{p}} = y - \omega\, \mathrm{Stk} - (1 - \omega)\, \mathrm{Stk} \exp (-t/\mathrm{Stk}) + \frac{1}{2} \mathrm{Stk}\, y (3 - y^2).$$

Concentration Distribution. To calculate the concentration distribution of the dispersed component we use the Eulerian and Lagrangian approaches.

Similar Solution. With account for (9), the continuity equation for the condensed phase will take the following form:

$$\chi \varphi + \psi \chi' + \psi' \chi = 0. \tag{19}$$

The boundary condition for Eq. (19) is set on the wall:

$$\chi (1) = \rho_{\mathrm{p,w}}.$$

Let us consider the behavior of the solution of Eq. (19) near the plane of symmetry. When $|y - 1| \ll 1$ this equation is written in the form

$$\psi_1 y \chi' + (\varphi_0 + \psi_1) \chi + o (y - l) = 0.$$

Using the expressions found earlier for φ_0 and ψ_1, we obtain

$$\chi (y) = \chi (l) \left(\frac{y}{l} \right)^{-(1 + \varphi_0/\psi_1)}.$$

Since $1 + \varphi_0/\psi_1$ is more than 0, the solution of Eq. (19) near the plane of symmetry is unbounded. This makes it difficult to use the self-similar representation of the solution of (10) in practice.

To avoid the nonphysical behavior of concentration we set $\rho_{\mathrm{p}} = 0$ for $x = 0$. Then we obtain $\rho_{\mathrm{p,w}}(x, 1) = \overline{\rho}_{\mathrm{p,w}}(x) \; \forall x \cup [0, \infty]$; here, we have $\overline{\rho}_{\mathrm{p,w}} = 0$ for $x = 0$ and $\overline{\rho}_{\mathrm{p,w}} = \rho_{\mathrm{p,w}}$ for $x = \infty$. One possible variant of selection of the function $\overline{\rho}_{\mathrm{p,w}}$ is that we set

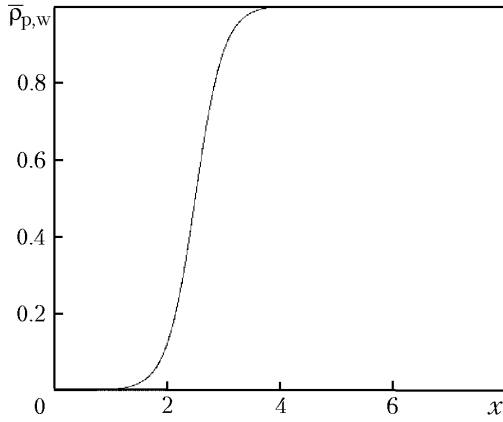$$\overline{\rho}_{\mathrm{p,w}} = \frac{1}{2} \left[1 + \tanh \left(\frac{x - x_0}{A} \right) \right],$$

Fig. 3. Distribution of the function $\bar{\rho}_{p,w}$ determining the boundary condition for the concentration of the condensed phase for $x_0 = 5$, $\omega = 1$, $\rho_{p,w} = 1$, Re = 10^3, and Stk = 10^{-3}.
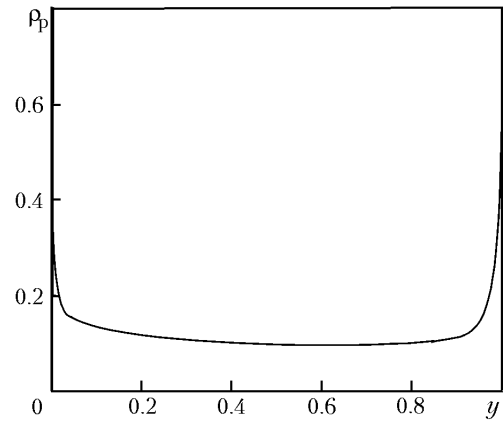
Fig. 4. Concentration distribution of the condensed phase for $x_0 = 1$, $\omega = 0.1$, $\rho_{p,w} = 1$, Re = 10^3, and Stk = 10^{-1}.

where $A \sim 10$. The distribution of the function $\bar{\rho}_{p,w}$ determining the boundary condition on the wall for the concentration of the condensed phase is shown in Fig. 3.

In connection with the fact that the solution of (19) is unbounded, the concentration of the dispersed phase is represented in the complete form (11) and is found by solution of the equation

$$\rho_p \varphi + x\varphi \frac{\partial \rho_p}{\partial x} + \psi \frac{\partial \rho_p}{\partial y} + \psi' \rho_p = 0 . \tag{20}$$

We show that the solution written in the form (10) may be considered as the limiting solution of Eq. (20) for $x \gg 1$. Let us consider the function $R(\xi, y) = \rho_p(x, y)$, where $\xi = 1/x$. For $x \gg 1$ we may write

$$R(\xi, y) = R_0(y) + O(\xi) .$$

Beyond the small vicinity of the line $y = 0$ where R is dependent on x, Eq. (20) will take the form

$$\psi R_0' + (\varphi + \psi') R_0 = 0 . \tag{21}$$

Equation (21) is identical to Eq. (19). Consequently, if we have $\bar{\rho}_{p,w} \to \rho_{p,w}$ for $x \to \infty$, the boundary condition for R_0 has the form $R_0(1) = \rho_{p,w}$. As a result, the problem for R_0 is identical to the problem of calculation of χ.

The concentration distribution of the condensed phase is given in Fig. 4 (the concentration has a nearly constant value in most of the computational domain ($\rho_{p,w} v_{p,w} \sim 0.1$)).

Exact Solution. Using the condition of equality of flow rates, we find the concentration distribution of the dispersed phase over the cross section of the channel:

$$\rho_p^o v_w dx_p = \rho_p u_p dy_p .$$

Passing to dimensionless variables, we obtain

$$\rho_p = \frac{2}{u_p} \frac{dx_p}{dy_p} .$$

When the solutions for channels with strong and weak injections are used, the condensed-phase distributions may be obtained in finite form (they are not given here because of the cumbersomeness of the corresponding relations).
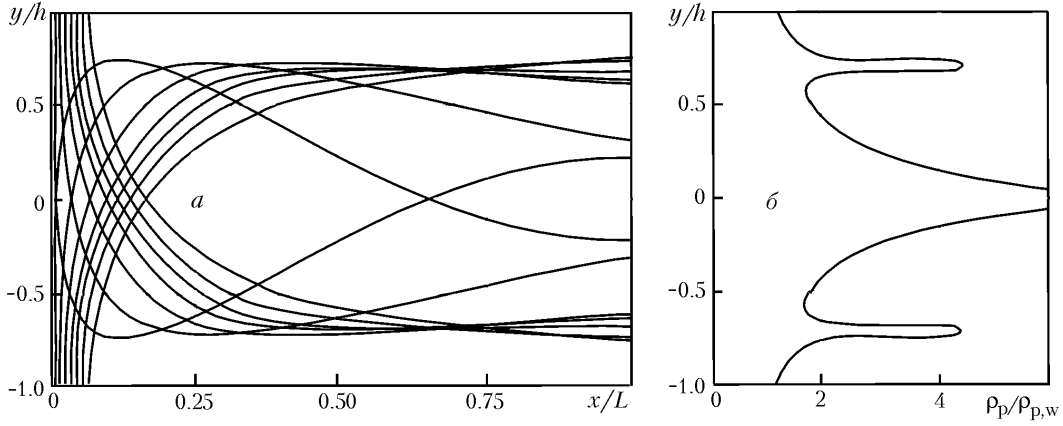
Fig. 5. Trajectories of a particle (a) and concentration distribution of the condensed phase (b) in a channel with a strong injection (Re $\to \infty$) for Stk = 1.8 and $\omega = 1$.

Order of Concentration Singularity. To investigate the singularity of the concentration of the impurity near the $y = 0$ line we use the continuity equation written in Lagrangian variables:

$$n_p (x_{p0}, y_{p0}, t) \frac{\partial y_p (x_{p0}, y_{p0}, t)}{\partial t} \frac{\partial x_p (x_{p0}, y_{p0}, t)}{\partial x_{p0}} + 1 = 0 . \qquad (22)$$

In the region of small particles (for 2π Stk < 1), the concentration of the dispersed phase has a singularity ($n_p(\mathbf{r}_p) \to \infty$ for $\mathbf{r}_p \to 0$). We denote a sphere of radius r with its center at $y = 0$ by $S(r)$. We compute the number of particles in this sphere:

$$N (r) = \int_S n_p d\mathbf{r} .$$

Using Eq. (22), we may show, analogously to [11], that

$$N (r) = C r^\gamma + o\, (r^\chi) .$$

The exponent γ determines the order of the concentration singularity of the dispersed phase ($1 + \sqrt{2} < \gamma < 3$ for plane flow). The concentration singularity of particles for $\mathbf{r}_p \to 0$ is integrable; it becomes weaker with increase in the particle size [11].

The trajectories of a particle and the concentration distribution of the condensed phase are shown in Fig. 5. The local maximum corresponds to the position of a separatrix that divides the computational domain into two subdomains: a vacant subdomain and that occupied by the particles.

Conclusions. Based on the parameter-expansion method, we have considered a number of problems associated with the modeling of two-phase flows in channels with a distributed injection. The distributions of the parameters of the condensed phase have been obtained accurate to the terms linear with respect to the Stokes number. We have singled out the factors exerting an influence on the velocity of nonequilibrium phase motion and have investigated the range of applicability of the solution obtained. The velocity and concentration distributions of the condensed phase in channels with a strong (nonviscous approximation) and weak (creeping flow) injection have been given in finite form. The possibility of applying such a solution to calculation of the concentration of the condensed phase has been shown.

The approach proposed and the solutions obtained may be applied to approximate evaluation of the parameters of gasdispersed flows in channels with injection and allow a comparatively simple generalization to the axisymmetric case.

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NOTATION

B, inertia parameter of a particle; $\mathbf{C}$, constant of integration; f and g, functions describing the dependences of the longitudinal and transverse components of the gas-phase velocity on the coordinate y; F, integral parameter; Fr, Froude number; h, channel half-width, m; H, ratio of the characteristic times; l, channel length, m; n, concentration, $1/m^3$; N, number of particles; p, pressure, Pa; q, exponent in the law of variation in the particle size; r, radius, m; $\mathbf{r}$, radius vector, m; Re, Reynolds number; S, spherical surface; Stk, Stokes number; t, time, sec; u and v, velocity components, m/sec; $\mathbf{v}$, velocity vector, m/sec; x, y, coordinates, m; γ, exponent; δ, ratio of the running particle size to the initial size; μ, dynamic viscosity, kg/(m·sec); ρ, density, kg/m^3; φ and ψ, functions describing the dependences of the longitudinal and transverse components of the dispersed-phase velocity on the coordinate y; χ, function describing the dependence of the concentration of the dispersed phase on the coordinate y; ω, coefficient of initial velocity nonequilibrium of phases. Subscripts and superscripts: b, burning; g, gas; m, maximum; p, particle; w, channel wall; o, true density; 0, initial instant of time; $'$, derivative with respect to the coordinate y; $\sim$, particular solution of a nonhomogeneous equation; $\wedge$, general solution of a homogeneous equation; $\bar{}$, boundary condition for concentration.

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